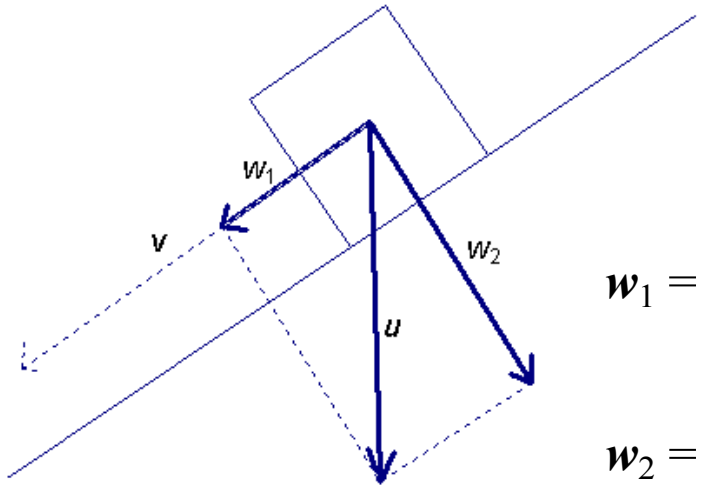
6-4 Vectors and Projections

$$\boldsymbol{w}_1 + \boldsymbol{w}_2 = \boldsymbol{u}$$

$$\boldsymbol{w}_1 = \text{proj}_{\boldsymbol{v}}\, \boldsymbol{u}$$



$$\boldsymbol{w}_1 = \text{proj}_{\boldsymbol{v}}\, \boldsymbol{u} = \left(\frac{\boldsymbol{u} \bullet \boldsymbol{v}}{\|\boldsymbol{v}\|^2} \right) \boldsymbol{v}$$

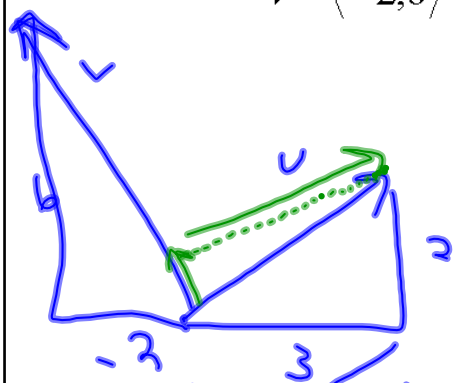
$$\boldsymbol{w}_2 = \boldsymbol{u} - \boldsymbol{w}_1$$

Ex 1 Find the projection of $\boldsymbol{u}$ onto $\boldsymbol{v}$. (Find $\boldsymbol{w}_1$)

Write $\boldsymbol{u}$ as the sum of two orthogonal vectors ($\boldsymbol{w}_1 + \boldsymbol{w}_2$).

$u = \langle 3,2 \rangle$

$v = \langle -2,6 \rangle$



$w_1 = \left(\frac{u \cdot v}{\|v\|^2} \right) v$

$= \left(\frac{6}{\sqrt{40}^2} \right) v$

$\frac{3}{20} \langle -2, 6 \rangle$

$\langle -\frac{6}{20}, \frac{18}{20} \rangle$

$w_1 = \langle -\frac{3}{10}, \frac{9}{10} \rangle$

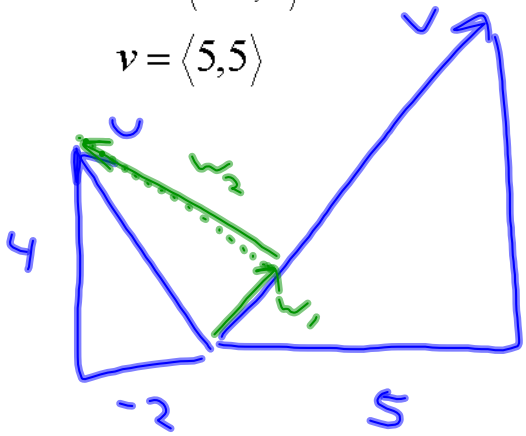
$w_2 = \langle 3.3, 1.1 \rangle$

Ex 2 Find the projection of $\boldsymbol{u}$ onto $\boldsymbol{v}$. (Find $\boldsymbol{w}_1$)

Write $\boldsymbol{u}$ as the sum of two orthogonal vectors ($\boldsymbol{w}_1 + \boldsymbol{w}_2$).

$u = \langle -2,4 \rangle$

$v = \langle 5,5 \rangle$



$w_1 = \frac{10}{50} \langle 5, 5 \rangle$

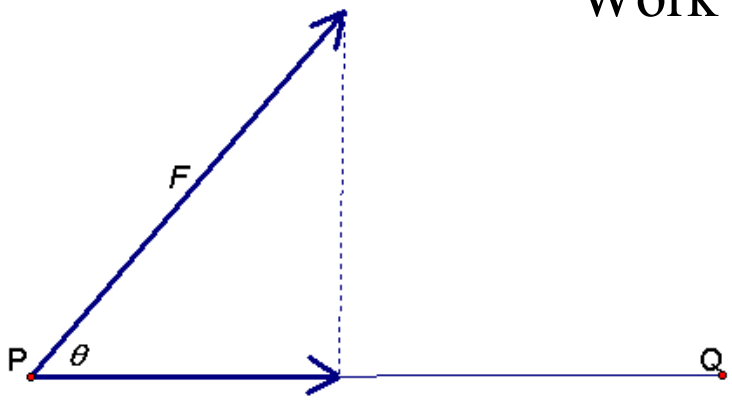
$5 = \langle 1, 1 \rangle$

$w_2 = u - w_1$

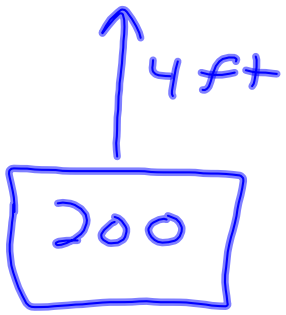
$w_2 = \langle -3, 3 \rangle$

Work = $\|F\|\|\overrightarrow{PQ}\|$

$$\text{Work} = \|\text{Proj}_{PQ}F\|\|\overrightarrow{PQ}\|$$
$$= \cos\theta\|F\|\|\overrightarrow{PQ}\|$$
$$= F \bullet \overrightarrow{PQ}$$



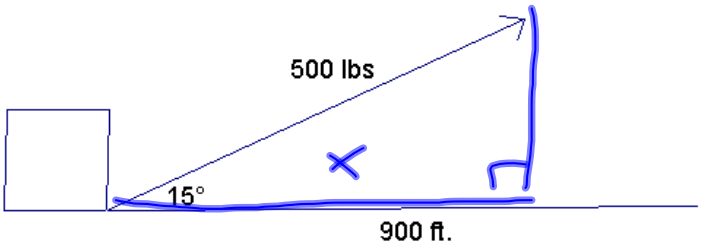
Ex 3 Find the amount of work needed to lift a 200 lb weight 4 feet.



(4 ft.)(200 lbs)

800 ft. lbs.

Ex 4 Find the amount of work needed if the force is 500 lbs and the distance is 900 feet.



$\cos 15^\circ = \frac{x}{500}$

$500 \cos 15^\circ = x$

$(482 \text{ lbs})(900 \text{ ft.})$

$433{,}000 \text{ ft lbs.}$

Ex 5 Find the work needed to move a particle from P to Q if the magnitude and direction of the force are given by $\boldsymbol{v}$.

$P = (3,5)$

$Q = (10,1)$

$v = \langle 3,1 \rangle$

$\langle 7, -4 \rangle \cdot \langle 3, 1 \rangle$

Homework
p.446
#45-48, 57-58